IDENTIFYING PATTERNS OF SKILLS ACQUISITION IN ELEMENTARY MATHEMATICS AMONG A COHORT GROUP OF PUPILS: IMPLICATIONS TO TEACHING AND LEARNING

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Abstract

This study explores the use of test performance to identify patterns of skills acquisition that differentiate good and poor performers in elementary mathematics. Good and poor performers in mathematics were identified through their cumulative raw score on six achievement tests in grade 1 to grade 6 mathematics. These tests were administered consecutively for six school years to an intact cohort group of 1,347 pupils towards the end of each school year. Discrimination and difficulty indices of all test items were computed to identify "critical" skills that highly discriminate the good performers from the low performers. The connection between the critical as well as non-critical skills in doing fractions and the patterns of acquiring these skills from one grade level to the next were then described and illustrated.

The results showed that majority of the critical items are more difficult than the non-critical items. The pattern of difficulty and discrimination indices of items on fractions indicated that both good and poor performers acquire the ability to identify fraction concepts from illustrations and perform addition and subtraction on similar fractions. Both groups, however, need to extend their conceptual understanding of fractions. The competencies of good and poor performers diverge at the point when they are required to compare fractions and execute basic operations on dissimilar and/or mixed form fractions. The results also showed how proficiency may be demonstrated with procedural knowledge without necessarily implying a good grasp of underlying concepts.

For many years, the Center for Educational Measurement (CEM) has been collecting and processing achievement test data in English, Mathematics, and Science at all levels of basic education for diagnostic purposes. Over 300,000 students from an estimated membership of at least 450 private and public schools take these tests annually before the academic year ends in March.

Although the databases of these tests have grown significantly from the time CEM was established, not much has been done to utilize the data to generate information that could systematically help build and create a knowledge base about what the Filipino learner learns and how he or she develops. This void prompted us to search for a cohort of pupils whose achievements in mathematics across six years of elementary education have been documented by our tests. It is hoped that this cohort study would yield the baseline data we need to be able to institute a viable research program on the cognitive development of the Filipino Learner.

OBJECTIVES

This study (1) explores the possibility of using performance in curriculum-aligned standardized tests to identify patterns of skills acquisition that differentiate good and poor performers in elementary mathematics and (2) attempts to look into implications of these patterns to teaching and learning by contrasting levels of good and poor performance.

METHOD

Selection of the sample

An intact cohort group of 1,347 grade 6 pupils from 11 schools was extracted from the database of CEM schools that subscribed to the Mathematics Diagnostic Tests for six consecutive school years. This group of pupils took the tests from grade 1 to grade 6. They entered first grade in 1996-97 with a mean age of about seven years, and finished elementary education in 2001-02 at more or less 13 years of age. A cohort group was the appropriate sample for this study because it allows us to consider developmental changes in skills acquisition from one grade level to the next.

Instruments

Acquired levels of knowledge and skills in elementary mathematics were measured using six standardized achievement tests in mathematics from grades 1, 2, 3, 4, 5, and 6; developed by CEM and administered consecutively for six school years to the same group of examinees towards the end of each school year. The test results are reported individually and by group (according to class section and school) in both the content and skills areas for each grade level prescribed by the national curriculum. The entire set of six tests consists of 320 multiple-choice items that are distributed as follows:

Content Area	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6	Total
Number Concepts & Numeration	12	12	8	4	4	4	44
Number Theory	…….	…….	4	8	…….	8	20
Addition	8	4	4	3	2	4	25
Subtraction	8	4	4	3	5	4	28
Multiplication	…….	4	4	5	5	4	22
Division	…….	4	4	5	4	4	21
Fractions	4	4	8	12	12	12	52
Decimals, Ratio, & Proportion	…….	…….	4	8	16	16	44
Geometry & Measurement	8	8	8	8	8	8	48
Graphs, Maps, & Scales	…….	…….	4	4	4	4	16
Entire Test	40	40	52	60	60	68	320

Organization and analysis of data

Individual responses of the pupils to all items of the six Mathematics Diagnostic Tests constitute the basic unit of analysis of this study. A correct or incorrect response to an item was scored as 1 or 0, respectively. Unanswered items, although computed as part of a cumulative score, were not included in the analysis. The analysis yielded the following statistics and classifications:

1. **Cumulative raw score.** This score is a summation of an individual student's scores in the mathematics tests from grade 1 to 6. It was computed and used simply to classify students as good or poor performers when their scores were arranged from highest (320) to lowest (0).

2. **Good vs. poor performers.** From the distribution of cumulative raw scores, the top 27% of the sample were labeled good performers while the bottom 27% were labeled poor performers.

3. **Item discrimination index (*D*).** To identify skills that separated the two groups, item discrimination indices were computed for all items. The discrimination index, *D*, of an item is computed as the difference between the proportions of good and poor performers who were able to answer the item correctly.

4. **Critical vs. non-critical items.** An item is labeled critical if *D* is *equal to or greater than 0.40*, and non-critical if *D* is *less than 0.40*. The critical items refer to skills that separate the cumulatively good and poor performers in elementary mathematics.

5. **Item difficulty index (*p*).** The difficulty index, *p*, of an item is computed as the proportion of pupils in the cohort group who were able to answer the item correctly. This index is used to ascertain which skills are relatively easy or difficult for the students to learn at a particular grade level.

LIMITATIONS OF THE STUDY

This study is archival, *ex post facto* research. A convenient sample was used, whose raw scores are not normally distributed but skewed towards the high scores, with a mean of about 65% correct answers. In addition, although the test instruments used are aligned with a prescribed curriculum, only the specific learning competencies deemed important by curriculum experts are measured by the final set of test items.

RESULTS AND DISCUSSION

This section focuses on (1) a description of the performance of the cohort sample on the six diagnostic tests; (2) the distribution and description of critical items in the content areas at each grade level; (3) a presentation of the difficulty levels of the test items; and (4) a discussion on the acquisition of skills in the area of fractions.

Test Performance Profile

The performance of the entire cohort is shown in Table 1. The minimum and maximum values at each grade level and for all levels combined indicate wide variation in scores from 15% to 100%. It must be noted that the mean total percent correct scores of the cohort tend to decrease as the group moved toward higher grade levels. Grades 4 and 5 showed the lowest mean scores. However, these scores went up again at grade 6. This tendency can be seen also with percentile points.

Table 1. Descriptive Information on Achievement Levels[1] of the Cohort Sample, n=1,347

Statistic		Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6	All Levels Combined[2]
Mean		75	69	60	57	56	64	63
SD		15	18	17	18	19	19	16
Minimum		18	15	15	18	17	19	27
Maximum		100	100	100	100	100	100	99
Percentile	25th	65	58	46	43	42	49	50
	50th	78	72	60	55	55	65	63
	75th	88	82	75	70	70	79	75

[1]Achievement is measured in terms of total percent correct scores at each grade level and for all levels combined
[2]Denotes total percent correct scores when raw scores for each grade level are summated and expressed as a percentage of the total number of items for all grade level tests combined

Comparing good and poor performers. Table 2 shows that mean scores of the good performers are higher than scores of the entire cohort. The percentile points indicate that approximately three-fourths of the group obtained scores at the relatively high end of the score range (greater than 75% correct). This is true for all levels except for grades 4 and 5.

It will be noted that at grade 5, performance of the good group showed wide variability. Scores range from 25%-100%, while in other grade levels, scores do not range wider than ~40%-100%. Around one-fourth of the good group scored between 25%-65% at grade 5, with approximately 10% scoring between 25%-40%.♣ This may indicate that the examinees found the grade 5 content more difficult than content in other grades.

Nevertheless, at each grade level, Table 2 shows that mean performance of the poor group is substantially lower (29-43 mean percentage points lower) than performance of the good group. For the poor group, mean total percent correct scores for grades 3 and above are less than 50%. Maximum scores of the poor group for grades 3 and 4 are also lower than in other grades. Percentile points indicate that approximately half of the examinees in the poor group scored within the range of 30%-50% for grades 3 and up.

Overall elementary mathematics performance is substantially different for the 2 groups. The summated scores for all grade levels for the high group yielded a mean total percent score of 83%. The bottom 25% in this group scored between 74%-77% correct. In contrast, the mean total percent correct score for the poor group for all the elementary mathematics tests combined is 43%. For this group, the bottom 25% scored between 27%-39% correct.

♣ Data not shown

Table 2. Descriptive Information on Achievement Levels of Good and Poor Performers[1]

	Statistic		Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6	All Levels Combined[2]
Good Performers, n=364	Mean		89	86	81	78	74	85	83
	SD		8	8	9	9	17	8	6
	Minimum		55	55	42	50	25	56	74
	Maximum		100	100	100	100	100	100	99
	Percentile	25th	85	82	75	72	65	81	77
		50th	90	88	81	78	75	86	82
		75th	95	92	87	85	88	91	87
Poor Performers, n=364	Mean		60	50	41	38	42	42	43
	SD		13	13	9	8	15	10	6
	Minimum		18	15	15	18	17	19	27
	Maximum		98	88	63	60	98	87	51
	Percentile	25th	52	40	35	32	32	34	39
		50th	60	50	42	37	38	41	44
		75th	70	58	46	43	47	49	48

[1]Achievement is measured in terms of total percent correct scores at each grade level and for all levels combined
[2]Denotes total percent correct scores when raw scores for each grade level are summated and expressed as a percentage of the total number of items for all grade level tests combined

Another way of characterizing the performance of the contrast groups is by comparing the mean difficulty levels obtained by these groups with those attained by the entire cohort and by the norm group for the diagnostic tests. As can be seen in Figure 1, mean *p* levels for each grade level test for the good performers were substantially higher than levels for the poor performers, the entire cohort and the norm group.

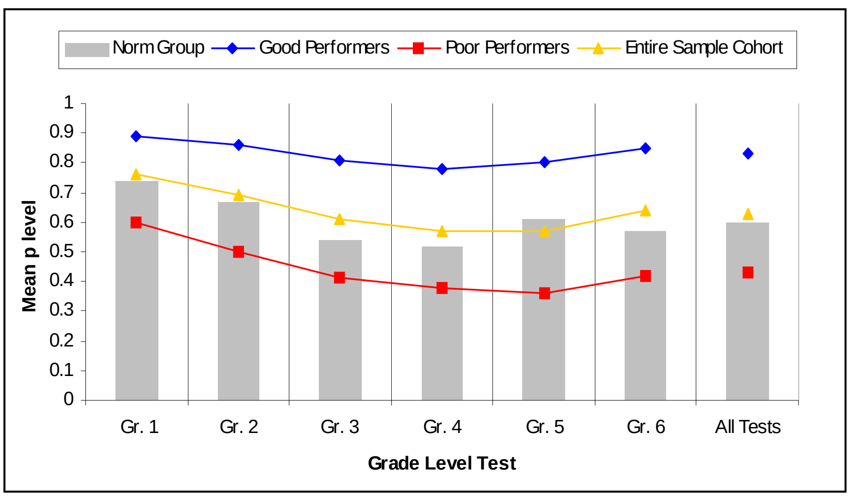


Figure 1. Comparison of Mean Difficulty Levels for each Test for the Contrast Groups, the Entire Cohort and the Norm Group

Items that discriminate between good and poor performance

<u>Content area.</u> The distribution of items per content area and grade level, and the proportion of these items that were identified as "critical" are presented in Table 3. From the table, it will be noted that the number of items increases with increasing grade level. However, even when expressed as a percentage of the total number of items per test, Table 3 shows that critical items increase with increasing grade level. This suggests a growing disparity between the skills learned by the good performers compared to the poor performers as they progressed through elementary level mathematics.

Table 3. Distribution of Critical Items at each Grade Level According to Content Area

Content Area		Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6	All Levels
Number Concepts & Numeration	No. of Items	12	12	8	4	4	4	44
	% Critical	*33%*	*17%*	*63%*	*75%*	*50%*	*50%*	*41%*
Number Theory	No. of Items			4	8		8	20
	% Critical			*75%*	*63%*		*25%*	*50%*
Addition	No. of Items	8	4	4	3	2	4	25
	% Critical	*38%*	*100%*	*0%*	*67%*	*50%*	*0%*	*40%*
Subtraction	No. of Items	8	4	4	3	5	4	28
	% Critical	*38%*	*100%*	*100%*	*100%*	*100%*	*50%*	*75%*
Multiplication	No. of Items		4	4	5	5	4	22
	% Critical		*50%*	*100%*	*100%*	*60%*	*75%*	*77%*
Division	No. of Items		4	4	5	4	4	21
	% Critical		*50%*	*75%*	*100%*	*100%*	*100%*	*86%*
Fractions	No. of Items	4	4	8	12	12	12	52
	% Critical	*0%*	*0%*	*0%*	*17%*	*83%*	*83%*	*42%*
Decimals and/or Ratio and Proportion	No. of Items			4	8	16	16	44
	% Critical			*25%*	*38%*	*63%*	*63%*	*55%*
Geometry & Measurement	No. of Items	8	8	8	8	8	8	48
	% Critical	*13%*	*25%*	*50%*	*13%*	*25%*	*75%*	*33%*
Graphs, Maps, and Scales	No. of Items			4	4	4	4	16
	% Critical			*25%*	*100%*	*50%*	*75%*	*63%*
Entire Test	No. of Items	40	40	52	60	60	68	320
	% Critical	*28%*	*40%*	*48%*	*55%*	*65%*	*62%*	*52%*

Also from Table 3 it will be observed that except for grades 3 and 6 *Addition* and grades 1-3 *Fractions*, all relevant content areas at each grade level yield critical items. For *Number Concepts and Numeration* and *Number Theory*, the percentage of critical items reach their highest for grades 3 and 4; for *Addition*, the highest percentage of critical items is at grade 2, while for the other operations, majority of items (more than 50%) are critical throughout the elementary period. For *Fractions* and *Decimals,* the highest percentage of critical items are at grades 5 and 6, for *Geometry and Measurement* at grades 3 and 6 and for *Graphs, Maps and Scales*, at grades 4 and 6.

Difficulty level. Table 4 shows the mean difficulty levels for each content area and item type at corresponding grade levels. As shown in Table 4, *p* values for each test indicate that generally, critical items are relatively more difficult than non-critical items. An exception is noted in grade 4, where mean *p* value for critical items is higher than non-critical items, indicating that critical items for this grade are relatively easier. However, when *p* values are inspected per content area, it will be noted that the relationship between item type and difficulty varies with content area. That is, for certain content areas, mean *p* values for critical items indicate that they are relatively easier than non-critical items or that the two item types have relatively similar difficulties. The former case is true for *Number Theory,* grade 3 *Division, Decimals,* and *Graphs, Maps and Scales*, grade 4 *Geometry and Measurement* and grades 4-6 *Fractions.* Difficulties for critical and non-critical items are comparable for grades 3 and 5 *Geometry and Measurement*, and grades 5-6 *Decimals, Ratio and Proportion*. For the rest, in general, critical items are more difficult than non-critical items. It will be noted from Table 4 that critical items do not occupy the extreme ends of the difficulty range for the entire test. These indicate that non-critical items may fail to discriminate between good and poor performers because the entire cohort as a whole found them too easy or too difficult.

Table 4. Mean Difficulty Levels at each Grade Level according to Content Area and Item Type*

Content Area	Item Type	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6	All Levels
Number Concepts and Numeration	*Critical*	*0.64*	*0.60*	*0.62*	*0.61*	*0.50*	*0.65*	*0.61*
	Non-Critical	0.76	0.71	0.69	0.88	0.84	0.81	0.75
	All Items	0.72	0.69	0.65	0.67	0.67	0.73	0.69
Number Theory	*Critical*			0.56	0.64		0.72	0.63
	Non-Critical			0.28	0.47		0.62	0.54
	All Items			0.49	0.57		0.65	0.59
Addition	*Critical*	*0.76*	*0.70*		*0.72*	*0.61*		*0.71*
	Non-Critical	0.85		0.80	0.90	0.95	0.81	0.84
	All Items	0.82	0.70	0.80	0.78	0.78	0.81	0.79
Subtraction	*Critical*	*0.67*	*0.57*	*0.55*	*0.64*	*0.74*	*0.62*	*0.64*
	Non-Critical	0.74					0.89	0.78
	All Items	0.71	0.57	0.55	0.64	0.74	0.75	0.67
Multiplication	*Critical*		*0.59*	*0.58*	*0.59*	*0.53*	*0.59*	*0.58*
	Non-Critical		0.71			0.82	0.80	0.77
	All Items		0.65	0.58	0.59	0.65	0.64	0.62
Division	*Critical*		*0.64*	*0.58*	*0.58*	*0.67*	*0.68*	*0.63*
	Non-Critical		0.78	0.38				0.65
	All Items		0.71	0.53	0.58	0.67	0.68	0.63
Fractions	*Critical*				*0.57*	*0.46*	*0.57*	*0.52*
	Non-Critical	0.78	0.80	0.70	0.55	0.42	0.45	0.64
	All Items	0.78	0.80	0.70	0.56	0.46	0.55	0.59
Decimals and/or Ratio and Proportion	*Critical*			*0.54*	*0.44*	*0.54*	*0.58*	*0.55*
	Non-Critical			0.33	0.52	0.55	0.59	0.52
	All Items			0.38	0.49	0.55	0.58	0.53
Geometry & Measurement	*Critical*	*0.65*	*0.59*	*0.57*	*0.65*	*0.42*	*0.64*	*0.59*
	Non-Critical	0.82	0.77	0.57	0.49	0.43	0.85	0.64
	All Items	0.80	0.72	0.57	0.51	0.43	0.69	0.62
Graphs, Maps, and Scales	*Critical*			*0.72*	*0.53*	*0.60*	*0.54*	*0.57*
	Non-Critical			0.70		0.68	0.66	0.69
	All Items			0.70	0.53	0.64	0.57	0.61
Entire Test	*Critical*	*0.68*	*0.62*	*0.58*	*0.59*	*0.56*	*0.60*	*0.59*
	Minimum**	0.58	0.46	0.36	0.40	0.31	0.36	0.31
	Maximum†	0.80	0.77	0.76	0.82	0.80	0.82	0.82
	Non-Critical	0.79	0.74	0.63	0.54	0.59	0.69	0.67
	Minimum**	0.45	0.36	0.20	0.18	0.20	0.33	0.18
	Maximum†	0.97	0.89	0.91	0.90	0.95	0.93	0.97
	All Items	0.76	0.69	0.61	0.57	0.57	0.64	0.63
	Minimum**	0.45	0.36	0.20	0.18	0.20	0.33	0.18
	Maximum†	0.97	0.89	0.91	0.90	0.95	0.93	0.97

*Item type refers to whether an item is "critical" or not.
**,† Shows the lowest and highest values respectively, not the mean values

Patterns of skills acquisition in the content area of Fractions

The highly discriminating items can be thought of as items where achievement gaps between good and poor performers in elementary mathematics are widest. They represent areas of variable performance, being neither the easiest nor the most difficult items, and hence are a potential source of insight on what skills make the difference between the two groups. With a longitudinal cohort group, it might also be traced at what grade level, and along what competencies the separation between the two groups begin. For purposes of illustration, the following discussion of acquired mathematics skills or competencies will focus on one particular content area: Fractions. This particular content area was chosen because it is consistently one of the lowest-scoring areas in elementary mathematics according to CEM data compiled from 1998-2003 (CEM, 2003).

The distribution of Fraction items in the diagnostic tests is shown in Table 5. Learning competencies evaluated by these items can be sorted into the following general categories: (1) *Fraction concepts,* (2) *Kinds of fractions*, (3) *Equivalence and ordinal relations*, (4) *Basic operations and word problem solving on fractions*. In Table 5, cells without entries indicate that there were no items evaluating that topic. This may be due to either one of 2 reasons: items are randomly selected for each test, and so not all prescribed learning competencies for a particular grade level may be evaluated in the test. Another reason is that the topic may not be part of test content for that grade, in alignment with the country's recommended curriculum from the Department of Education. Therefore it may also be the case that competencies that are supposed to be acquired at a previous grade are not included for assessment in diagnostic tests for succeeding grades.

Table 5. Distribution of Fraction items by Learning Competencies

Learning Competency	Number of Items					
	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	Gr. 6
Fraction concepts	4	4	2	---	---	---
Kinds of fractions	---	---	---	2	1	2
Equivalence and ordinal relations	---	---	2	1	1	2
Basic operations and word problem solving						
Addition and subtraction of fractions	---	---	4	4	4	3
Multiplication of fractions	---	---	---	3	3	1
Division of fractions	---	---	---	2	3	4

Table 6 shows the difficulty levels of specific learning competencies evaluated at each grade. Whenever possible, items tapping comparable learning competencies across grade levels are aligned horizontally with each other. These learning competencies will be discussed by category in the following sections.

Fraction concepts (Grades 1-3). Children's initial understanding of fraction concepts is established through pictures or models. Typical illustrations used to expound these fraction concepts are whole objects separated into parts, or sets of objects where the objects are divided into smaller subgroups. From grades 1-3, the concepts are explored using such figures, moving from even-numbered to odd-numbered fraction parts, i.e., $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$. For items assessing comprehension of such visual representations of fractions, discrimination indices were not high enough (D<.40) to distinguish overall good and poor math performers. Mean difficulty levels for the entire cohort on these items indicate that these are ideas easily grasped by elementary students, except in 2 items which they found relatively difficult. In Table 6, these are items 20 and 27 for the grade 1 and 3 tests respectively. Unlike the other items which asks for the fractional part of the set that is indicated by shaded objects, these two items use unshaded objects and asks the student to indicate how many objects comprise a particular fractional notation (see Figure 2).

Table 6. Mean Difficulty Levels (*p*) of Critical and Non-critical Fraction Items from Grades 1 to 6

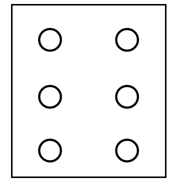
	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
Fraction concepts	Item# 26; ***p***=.90 Recognizes 1/2 and 1/4 of a whole					
	Item# 4; ***p*** =.85 Separates a whole object into fourths	Item# 12; ***p*** =.76 Separates a whole object into sixths				
	Item# 27; ***p*** =.80 Separates a whole object into thirds	Item# 28; ***p*** =.85 Separates a whole object into sixths	Item# 29; ***p*** =.90 Separates a whole object into thirds			
	Item# 20; ***p*** =.57 Separates a group of objects into halves and fourths	Item# 30; ***p*** =.79 Identifies 1/3 of a given group of objects	Item# 27; ***p*** =.20 Finds the fractional part of a set			
		Item# 6; ***p*** =.81 Identifies 1/5 of a given group of objects				
Kinds of fractions				Item# 3; ***p*** =.82 Identifies improper fractions from a given set of fractions		Item# 64; ***p*** =.44 Identifies improper fractions from a given set of figures
				Item# 47; ***p*** =.58 Identifies similar fractions from a given set of fractions	Item# 7; ***p*** =.69 Identifies similar fractions from a given set of fractions	Item# 57; ***p*** =.46 Identifies dissimilar fractions from a given set of fractions
Equivalence and ordinal relations			Item# 40; ***p*** =.64 Compares unit fractions			Item# 27; ***p*** =.48 Identifies equivalent fractions
			Item# 1; ***p*** =.85 Orders fractions less than one or equal to one (similar fractions)	Item# 60; ***p*** =.28 Orders dissimilar fractions in simple form	Item# 59; ***p*** =.31 Orders dissimilar fractions in simple form	Item# 18; ***p*** =.64 Orders dissimilar fractions in mixed form
Addition and subtraction of similar fractions				Item# 16; ***p*** =.56 States a principle involved in adding fractions		
			Item# 30; ***p*** =.78 Adds similar fractions with denominators in a range of 1 to 3			
			Item# 9; ***p*** =.74 Adds similar fractions with denominators in a range of 4 to 6	Item# 15; ***p*** =.63 Adds similar fractions		
				Item# 36; ***p*** =.70 Adds mixed numbers with similar fractions		
			Item# 33; ***p*** =.72 Subtracts similar fractions with denominators in a range of 6 to 10	Item# 31; ***p*** =.86 Subtracts similar fractions		Item# 67; ***p*** =.47 Subtracts similar fractions in mixed forms
			Item# 6; ***p*** =.80 Subtracts similar fractions with denominators in a range of 11 to 15			

Table 6. Mean Difficulty Levels (*p*) of Critical and Non-critical Fraction Items from Grades 1 to 6

	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
Addition and subtraction of dissimilar fractions					Item# 56; ***p***=.34 Adds mixed forms (dissimilar fractions)	Item# 29; ***p***=.63 Adds dissimilar fractions in simple or mixed forms
					Item# 44; ***p***=.49 Subtracts dissimilar fractions	Item# 16; ***p***=.68 Subtracts dissimilar fractions in mixed forms
					Item# 27; ***p***=.38 Subtracts mixed form from whole numbers	
					Item# 22; ***p***=.42 Solves 1-step word problems involving subtraction of fractions	
Multiplication of fractions					Item# 21; ***p*** =.53 Multiplies similar fractions in simple or mixed form	
				Item# 7; ***p*** =.82 Multiplies dissimilar fractions	Item# 57; ***p***=.37 Multiplies dissimilar fractions in simple or mixed form	Item# 49; ***p***=.62 Multiplies whole numbers by fractions
				Item# 26; ***p*** =.40 Solves word problems involving multiplication of a fraction and a whole number		
				Item# 59; ***p*** =.30 Solves word problems involving multiplication of a mixed fraction and a whole number	Item# 55; ***p***=.41 Solves word problems involving 2 or more fundamental operations on fractions	
Division of fractions				Item# 28; ***p***=.30 Divides a whole number by a fraction		Item# 30; ***p***=.57 Divides whole numbers by fractions
				Item# 55; ***p***=.41 Divides similar fractions	Item# 47; ***p***=.55 Divides a fraction by another fraction	Item# 66; ***p***=.56 Divides fractions by another fraction
					Item# 20; ***p***=.54 Divides a fraction by a whole number	
					Item# 24; ***p***=.55 Solves 1-step word problems involving division of fractions	Item# 59; ***p***=.53 Solves 1-step word problems involving division of fractions
						Item# 32; ***p***=.48 Solves 2-step word problems involving multiplication and division of fractions

*Items in red are critical items

More than half (.57) of the cohort correctly answered the grade 1 item, but, only 2 out of every 10 students (.20) did for the grade 3 item[ϕ]. Particular difficulties with this question may be conjectured. One is that there are no visual cues to the answer. On items where the fractional part is indicated with shading, children can simply count how many parts or objects are shaded, how many parts or objects there are in all, and choose the appropriate fraction notation whose numerator and denominator respectively, correspond to the counted amounts. For the given stimulus figure, a bit of flexibility in fraction understanding is required. Some strategies that children have been observed to use in experiments (involving manipulatives in the form of set models) were to "dole out" or divide the objects among recipients equally until all the objects have been given, and then count how many objects each recipient had. Other strategies include using multiplication or division knowledge to figure out the answer (Jensen, 1993).



Grade 1: Item 20; p = .57: *Which numeral shows 1/2 of the number of circles in the set above?*

Grade 3:Item 27; p = .20: *What is 1/3 of the number of marbles above?*

Figure 2. Sample stimulus and stems of items assessing understanding of fractional parts of sets (difficult)

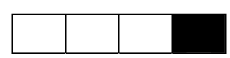
Another feature that makes these two items more difficult than others tapping similar competencies is that the number of objects in the illustrated set, 6 objects, does not correspond to the number in the denominator of the fractional notation the children are being asked to express: $\frac{1}{2}$, $\frac{1}{3}$. Studies of children 9 years old and below have shown that dividing objects into fair shares is easiest to understand when the number of objects in the set is equal to the number of shares being asked (Jensen, 1993). This explanation, together with the help of visual cues (shading), is borne out by the ease which students found in answering the items illustrated in Figure 3.

Kinds of fractions (Grades 4-6). At grade 4, introduction to more formal labels of fraction concepts begins. Terms such as similar, dissimilar, proper, improper, and mixed form fractions are defined. As can be noted from Table 6, grade 4 and 5 competencies that relate to identifying kinds of fractions (specifically similar fractions) discriminate between good and poor overall math performers.

Addition and subtraction can only be directly done on similar fractions, thus, recognizing similar fractions or converting fractions into such a form is an essential step. Much of the later work on fractions is on calculation and problem solving and clarity of the similar fractions concept may be important for properly carrying out certain operations.

While the grade 4 and 5 items on identifying similar fractions clearly discriminated between overall good and poor math performers, (approximately 8-9 out of every 10 good performers correctly answering versus only 3-4 out of every 10

Grade 1: Item 4; p=.85
What part of the rectangle is shaded?



(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Grade 2: Item 30; p=.79

Which of the following sets shows that $\frac{1}{3}$ *of it is shaded?*

(A)
(B)
(C)
(D)

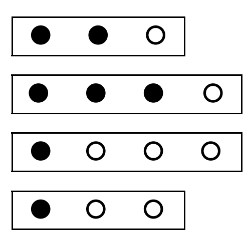


Figure 3. Sample items for assessing understanding of fractional parts of sets (easy)

[ϕ]Based on difficulty levels for the good and poor performers. Data not shown.

poor performers)[ϕ] it will be noted that the grade 6 item on the opposite concept (identifying dissimilar fractions), indicated relatively greater difficulty for the entire cohort, with only 6 out of 10 good performers answering correctly (see grade 6 item number 7 in Table 6, however this is still considerably higher than the proportion in the group of poor performers who answered the item correctly, 3 out of 10)[ϕ].

The grade 4 item on identifying improper fractions on the other hand, is a relatively easy task for the entire cohort. Approximately 8 out of every 10 students correctly answered it[ϕ]. The grade 6 item on the same competency however, was relatively more difficult for the cohort. The item asks the student to indicate which set of shaded figures correctly represents an improper fraction. Even among good performers, only half correctly answered the item[ϕ] (see Table 6). Again this may have to do with the students not yet fully grasping the meaning of this fraction type hence failing to see its figural representation.

Establishing equivalence and ordinal relations–comparing and ordering fraction values (Grades 3-6). Understanding fraction sizes in relation to each other is another important and often difficult concept for elementary students. Often, size relations among fractions seems counterintuitive to size relations established with whole numbers: 5 is *greater than* 3 but $\frac{1}{5}$ is *less than* $\frac{1}{3}$. In this respect, comparing sizes of similar fractions probably follows a reasoning process that is closer to whole number thinking and thus might be easier for those beginning to study fractions.

This does seem to be the case with the present data. Note from Table 6 that item 1 under the Grade 3 column, which required sequencing similar fractions according to magnitude, are associated with relatively low difficulty. Eight out of every 10 students in the entire cohort correctly answered this item. For item 40 under the grade 3 column, while the fractions being compared are not similar, it is supposed that the relative sizes of unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$) is a basic lesson that is often rehearsed and thus becomes memorized fact. Six out of every 10 students in the cohort correctly determine relative sizes of unit fractions.

For succeeding grade levels, the introduction of dissimilar fractions into the sequencing activity increases the difficulty of the task. At grades 4 and 5, only 3 out of every 10 students in the entire cohort correctly sequence dissimilar fractions according to magnitude. However, by grade 6, the competency becomes a discriminating item, indicating that distinctly more good performers successfully complete such a task (8 out of every 10 good performer versus 4 out of every 10 poor performer)[ϕ]. Similarly, proficiency at identifying equivalent fractions at this grade distinguishes good from poor performers.

Performing basic operations and problem-solving in fractions (Grades 3-6). As can be seen in Table 6 under the grade 3 and 4 columns on addition and subtraction of similar fractions, good and poor performers' skills at these tasks do not differ greatly. Difficulty levels for these items at grades 3 and 4 indicate that the entire cohort found the tasks relatively easy ($.63 \leq p \leq .86$). A curious observation at grade 3 is the facility shown by the pupils for computational algorithms used to solve fractions ($.72 \leq p \leq .80$ for computation items), although they had a fairly weak understanding of some fraction concepts (grade 3, item 27, $p = .20$).

The ability to recognize a principle involved in adding fractions, that they must be similar in form, is a critical item at grade 4 (see grade 4, item 16). This recognition facilitates carrying out the correct procedure when operating on fractions (see items 15, 31 and 36). Multiplication at grade 4 (excluding word problem solving), is also relatively easy for the entire sample ($p = .82$), perhaps because the fractions being multiplied for that particular item are unit fractions, and so requires a relatively simple procedure to answer. Division of fractions and problem-solving at grade 4 are relatively difficult for the whole cohort ($.30 \leq p \leq .41$).

At grades 5 and 6, good and poor performers' competencies at basic operations on fractions become more distinct. Note from the grades 5 and 6 columns of Table 6 that more items become critical at these grade levels

[ϕ]Based on difficulty levels for the good and poor performers. Data not shown.

(10 out of the 12 items pertaining to fractions), and typically involve performing operations and problem-solving on fractions in dissimilar and/or mixed forms.

FINDINGS AND IMPLICATIONS

Based on the pattern of difficulty levels and discrimination indices on this sample, both good and poor performers show the ability to identify fraction concepts when presented in fairly simple forms and with visual cues. Both groups however, need to extend their conceptual understanding of fractions. Both groups also competently perform addition and subtraction on similar fractions. However, the competencies of good and poor performers diverge at the point when they are required to compare values and execute basic operations on dissimilar and/or mixed form fractions.

Thus, as may rightly be expected, tasks that impose greater cognitive demands (i.e., involve more procedural steps) are what separate good and poor performers in elementary mathematics. However, an unexpected though related finding also points to the fact that demonstrating proficiency on such procedures does not necessarily imply a good grasp of the underlying concepts. The implications of these results are further drawn out in the following discussion.

Implications to Mathematics Learning and Instruction

The topic of fractions is usually an area of difficulty for most students. For one, children are not typically exposed to the use of fraction labels in everyday language. Whole numbers are more familiar. Moreover in terms of notation, size relations in fractions seem to contradict what is learned with whole numbers, i.e. while 5 is bigger than 3, 1/5 is smaller than 1/3. The procedures for performing basic operations on fractions are also more complicated than those for whole numbers. For addition and subtraction, only the numerators are operated on, but only if the denominators are similar; to make fractions similar, steps involving division and multiplication have to be carried out. In multiplication, both numerator and denominator are multiplied, while in division, the inverse of the divisor is obtained and is actually *multiplied* to the dividend.

These difficulties involved in learning mathematics have been explained from both constructivist and information-processing perspectives (Byrne, 2001). When children move from studying whole numbers to studying fractions, they find that many of their previous *schemata* or "mental templates" if you will, for understanding numbers no longer work. The confusion in understanding fraction size relations is an example of this. There is a need to reconstruct the number system in the child's mind, and often this is done by relating fractional notations to concrete figures or collections of objects, to impart the initial understanding that fractions represent parts of wholes.

Yet, while instruction may start with such a purpose in mind, how well is this understanding conveyed? Is it imparted at all? The use of figures to illustrate concepts is done primarily in the early grades and instruction moves on to emphasize techniques for calculation. In the present study, there is some evidence to say that at higher grades conceptual understanding of fractions may still not be well developed, although ability to carry out procedures for computation may indicate mastery. This finding has been noted in other studies (Goldin & Passantino, 1996), and raises the question, what are we really teaching our students about math? And what does it indicate of the system that a student may progress to higher levels of study without really understanding some basic concepts?

A constructivist approach to teaching and learning would emphasize the need to relate new information to experience and previous knowledge, to facilitate reorganization of the child's current understanding of a topic. At the elementary level, a crucial concept that might do well to bridge the understanding between whole numbers and rational numbers (fractions) is the idea of numbers as comprising other numbers; even whole number notations express a quantity that can be thought of as being made up of smaller parts. This has been termed the *part-whole schema*, and is considered "an important breakthrough in mathematical development" (Jensen, 1993). For example, the notation *5* denotes not just the size of a set, but may also describe a set consisting of smaller sets of *3* and *2* members. Now certainly, such thinking about whole numbers is closer to the part-whole meaning of fractions (Jensen, 1993).

Nevertheless, while it makes intuitive sense to expect that enriching conceptual knowledge will enhance memory for procedural knowledge, studies have shown this is not always the case (Byrne, 2001). Even with instruction that is oriented toward a richer conceptual understanding of numbers and why certain procedures are more appropriate for solving certain problems than are others, students may continue to employ the wrong procedures, simply because they cannot remember the steps for the correct one. The information-processing approach would explain this as error in "retrieval" or recollection of the correct information, and the simple, straightforward strategy of "practice makes perfect" is still the solution (Byrne, 2001).

There are numerous benefits to linking conceptual and procedural knowledge. Mastery of concepts can help in recalling and applying procedures correctly, whereas well-learned procedures can help in building new concepts. Certainly, mathematics competency would not be complete if either were deficient (Hiebert, 1986). Instruction should build on both, in order for the student to have a truly sound understanding of mathematics. For the present study, results suggest the need for instruction that would further develop students' grasp of elementary fraction concepts.

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